Why Maximizing Information Ratios is Incorrect

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Abstract

Many papers on active management argue for maximizing information ratios using a risk-budgeting framework. Recent innovations in risk-adjusted performance measures show why maximizing information ratios could be the wrong policy and also provide a different twist to the discussion on separating alphas from betas. The literature on maximizing information ratios focuses only on the active management process and ignores two actions used by clients or managers to improve risk-adjusted performance: passive management and leverage/deleverage using cash. This paper demonstrates the impact of maximizing the wrong objective function and shows the benefit of maximizing risk-adjusted returns for the entire fund, rather than the information ratio on the active component.
Introduction

Since the prospective returns for passive benchmark are moderate (having declined over the last three years), clients are searching for higher returns. Many of them are exploring different ways to generate excess returns over the benchmark (alpha) through a mix of strategies. However, the challenge for clients is to construct robust portfolios using the many possible investment ideas that they have researched or have access to. All of this, however, needs to be done within existing risk budgets.

Portfolio managers are trying to create portfolios of the best investment rules for their clients to either hold onto or gain new assets. Pension funds are trying to implement the best active management decisions (tactical asset allocation, sector allocation etc.) and hire and allocate appropriate amounts to the best external managers (where portfolios are managed by external managers). In essence, risk management today is about making the best investment decisions to generate the highest possible returns per unit of risk.

There have been a flood of papers on the fundamental law of active management and how clients should select optimal portfolios. In their widely read book, Grinold and Kahn (1999) make three distinct claims that are disputed in this paper (and related research): (i) “Larger information ratios are better than smaller,” (page 6); (ii) “The notion of success is captured and quantified by the information ratio,” (page 110); and (iii) “every investor seeks the strategy or manager with the highest information ratio,” (page 125). This book and the papers that followed make the assumption that investors wish to maximize the
information ratio (defined as the excess return over the benchmark divided by the standard deviation of excess returns) or alternatively, maximize the return over benchmark for a target risk budget (called the tracking error budget) by optimally allocating across many competing active alternatives. The idea behind this approach, not always explicit, is that investors make a choice between how much of the portfolio will be active versus passive (benchmark performance), and then try to maximize the returns on the active portion for the given risk budget. However, by doing so they are creating inefficient portfolios either because they have created the portfolio in two stages or have excluded useful investment options that can allow investors to maximize risk-adjusted returns. The idea of using investments in risk-free assets or passive benchmarks to increase risk-adjusted returns have been examined in the context of risk-adjusted performance measures and increasingly such performance measures are providing guidance on portfolio construction. In short, investors should try to maximize risk-adjusted returns, not just information ratios on the active portfolio.

**Why Information Ratios are Incomplete Statistics**

For the longest time, investors satisfied themselves with using the Sharpe ratio (ratio of excess return over the risk-free rate to the standard deviation of that excess) and the Information Ratio (ratio of excess returns vis-à-vis a benchmark to standard deviation of excess returns) to distinguish whether risk-adjusted performance was good or poor. The standard deviation of excess returns is referred to as tracking error in the asset management industry and the formula for tracking error and information ratio is provided
in the Appendix 1. The simplest reason for disregarding the strategy with the highest information ratio is that it may not produce adequate returns if the risk taken is too low. For example, a strategy with an information ratio of 1, but which takes only 0.1% risk generates only 0.1% of returns. We move beyond this simple shortcoming to show that even in the context of full utilization of risk budgets, it could potentially be an inadequate measure.

A major contribution by Modigliani and Modigliani (1997) highlighted that such measures were inadequate for investors as the ratios said nothing about performance and provided no guidance as to how portfolios should be constructed. After all, one cannot use a ratio to pay pensioners or any other liability, whereas risk-adjusted performance is a more direct measure of success.

By adapting the Sharpe ratio, they were able to provide a true performance measure (i.e., measured in basis points of performance) that adjusted for differences in volatility between a portfolio and its benchmark. This measure also provided some guidance as to how much should be invested in the risk-free rate and the portfolio being reviewed. This measure came to be called the $M^2$ measure of risk-adjusted performance and the authors were able to demonstrate that rankings of portfolios would be changed when this measured was used as opposed to raw performance. They were able to also show the flaws in using the information ratio – a measure that potentially rejected portfolios with high risk-adjusted performance because they had negative information ratios. Research by Muralidhar (2000) extends this approach to also include the possibility of adding
passive portfolios to the optimal mix (i.e., in addition to the risk-free asset and the active opportunities) to demonstrate that this allowed for an optimal normalization of risk-adjusted excess returns subject to a tracking error budget. More recent work (Muralidhar 2002) shows that information ratios may not say enough about confidence in skill as high information ratios could lead to low confidence in skill depending on how the performance has been generated (i.e., requires an examination of the excess returns, the variances of the benchmark and active strategy and the correlation between the two). In addition, all these measures suffer from the problem of not adjusting for time, as a high information ratio over a short period of time may convey the same confidence in skill as a much lower information ratio over longer periods of time. For this paper, we ignore the time dimension and return to it in future research.\textsuperscript{v}

Therefore, investors who seek to maximize the information ratio over a number of different active strategies or managers run a big risk – of excluding the possibility of two basic alternatives, namely using the risk-free asset (or cash) and passive alternatives to create portfolios with the optimal risk-adjusted excess returns. None of these possibilities is out of the realm of usage either by an active manager or within an active strategy and hence it shows how a simple, naïve assumption of excluding investment possibilities can lead to the wrong result.\textsuperscript{vi}

\textbf{The M$^2$ Measure}
Modigliani and Modigliani (1997) make an important contribution by showing that the portfolio and the benchmark must have the same risk to compare them in terms of basis points of risk-adjusted performance. They define risk as the volatility or standard deviation of a portfolio’s returns. They propose that the portfolio be leveraged or deleveraged using the risk-free asset. In effect, combining cash with the active management strategy creates a more effective portfolio. If $B$ is the benchmark being compared to portfolio $I$, $r$ denotes the return, and $\sigma$ denotes the standard deviation, the leverage factor, $d$, is defined as follows:

$$d = \frac{\sigma_B}{\sigma_I} \quad (1)$$

The $M^2$ adjustment creates a new portfolio, called the risk-adjusted portfolio (RAP), whose return, $r(RAP)$, is equal to the “leverage” factor multiplied by the original return plus one minus the leverage factor, multiplied by the risk-free rate. Thus, if portfolio $F$ is the risk-less asset with zero standard deviation and is uncorrelated with other portfolios, the risk-adjusted return

$$r(RAP) = d*r(\text{active portfolio}) +(1-d)r(F), \quad (2)$$

where $\sigma_{RAP} = \sigma_B$  \quad (3)

Figure 1 demonstrates this transformation by carving up the return-standard deviation space into four regions and showing how a strategy may be levered or delivered to have
the same standard deviation as that of the benchmark. The correlation of the original portfolio to the benchmark is identical to the correlation of the RAP to the benchmark, as “leverage or deleverage” using the risk-free rate does not change the correlation characteristics (because of the zero correlation). The correlation is normally less than unity. If the correlation = 1, it could lead to a riskless arbitrage.

<INSERT FIGURE 1 HERE>

Specifically, the four regions in Figure 1 using the $M^2$ measure are where an investor would experience:

I. Portfolio outperformance on an absolute and risk-adjusted basis (positive information ratio).

II. Portfolio outperformance on an absolute basis and underperformance on a risk-adjusted basis (positive information ratio).

III. Portfolio underperformance on an absolute and risk-adjusted basis (negative information ratio).

IV. Portfolio underperformance on an absolute basis and outperformance on a risk-adjusted basis (negative information ratio).

The paper suggests that this $M^2$ adjustment allows for a comparison of “apples to apples”, namely, returns from the benchmark and the RAP have the same volatility. It shows that
making this adjustment can reverse peer rankings of mutual funds, managers or active strategies, especially those based on the Information Ratio. As a result, the paper discards the use of the information ratio as it could lead to incorrect decisions. For example, portfolios in Region IV would have a negative information ratio, but would have a risk-adjusted performance greater than the benchmark, or one would accept a manager in region II who has a negative risk-adjusted performance. This first step in adjusting for risk is adequate to negate the three major claims of Grinold and Kahn (1999) that were highlighted in the introduction. The $M^2$ measure is also preferred to the Sharpe Ratio as it expressed risk-adjusted performance in terms of basis points of outperformance and provides guidance on assets allocated to the active asset allocation strategy or external manager (allocation of $d$) and the risk-free asset (allocation of $1-d$). Graham and Harvey (1997) propose a variation of this method, assuming the riskless asset need not be an asset uncorrelated with other assets. This only leads to different allocations across funds rather than suggesting a new approach.

The $M^2$ adjustment made the comparison in terms of basis points of outperformance by ensuring all portfolios had the same variance as the benchmark. However, it ignored the possibility of investing in the passive benchmark. As a result, the one major shortcoming was that two investment strategies, normalized for the benchmark volatility, could have different correlations with the benchmarks and hence different tracking errors (see equation (A.1)). Tracking error is important to investors, especially institutional investors or those to whom responsibility has been delegated (i.e., in pure economic parlance, principals who have hired agents to manage their portfolios) because it provides a
measure of the variability of a strategy/manager’s returns around the benchmark. Investors would prefer, all else being equal, funds/agents with lower tracking error (and hence greater predictability in returns and higher confidence in skill). Hence the M² rankings could provide investors with incorrect information about the relative risk-adjusted performance of funds for certain types of investors.\textsuperscript{vii}

\textbf{The M³ methodology – Adjusting for Differences in Correlations}

An investor has to rely on available data to make projections for the future. Assuming historical distributions are preserved in the future, the three-dimensional problem of a comparisons of return, standard deviations, and correlations ($\rho$) has to be synthesized into a simple two-dimensional space of return and risk.\textsuperscript{viii} In mean-variance space, the risk-less asset is portfolio $F$ (with returns $r(F)$) and it can be used to leverage or deleverage the desired mutual fund/manager. In tracking error space, the only risk-less portfolio is the one with zero tracking error or the benchmark portfolio as it is perfectly correlated with itself (where $\rho = 1$, TE $= 0$, as $\sigma_B = \sigma_1$). Therefore, combining active strategies/managers with passive benchmarks and the risk-less asset can be used to alter the overall portfolio’s standard deviation and its correlation with the benchmark.

To create measures of correlation-adjusted performance, the investor needs to invest in the active strategy (or strategies), the risk-less asset and benchmark to ensure: (a) the volatility of this composite is equal to that of the benchmark (Modigliani and Modigliani 1997); and (b) the tracking error of this composite is equal to the target tracking error
This new measure, termed the $M^3$ measure, recognizes that the investor has to consider basis points of risk-adjusted performance after ensuring that \textit{correlations of various funds versus the benchmark are also equal}. In effect, now all tracking errors are normalized to the same value.

\textit{The $M^3$ model as Applied to a Retail Investor}

While the principle is identical for an investor hiring an external manager or implementing an active management strategy (i.e., an investment manager), this section assumes that an investor is invested in a Defined Contribution pension plan and must evaluate several mutual funds. Hammond (1997) states that to establish performance-related thresholds for managers, the investor must set a target tracking error and compare funds to the target. A similar approach is proposed by Litterman, Longerstaey, Rosengarten and Winkelmann (2001).

Assume that the investor is willing to tolerate a certain target annualized tracking error around the benchmark, say 300 bps ($TE(\text{target})$). The investor essentially wants to earn the highest risk-adjusted alpha for given tracking error and variance of the portfolio. Now define $a$, $b$, and $(1-a-b)$ as the proportions invested in the mutual fund, the benchmark, and the risk-less asset. Let $CAP$, be the correlation adjusted portfolio and therefore the returns of a $CAP$,

$$r(CAP) = a*r(\text{mutual fund}) + b*r(B) + (1-a-b)*r(F) \quad (4)$$
As one can see, this is an extension of the $M^2$ measure. Further, the investor must hold appropriate proportions of each to ensure the final portfolio has the target tracking error and the standard deviation of the benchmark. For a specific mutual fund, say mutual fund $I$, with a risk adjusted return $r(CAP-I)$, equation (4) can be re-written as:

$$r(CAP-I) = a*r(1) + (1-a-b)*r(F) + b*r(B) \quad (4')$$

where the coefficients of each portfolio represent the optimal weight of that specific portfolio to ensure complete risk adjustment. In addition, from the constraint on tracking error, there is a unique target correlation between the CAP and benchmark $B$. As demonstrated in Muralidhar (2000), this target correlation of the portfolio with that of the benchmark ($\rho_{T,B}$) is given by the equation for tracking error (equation (A.1)) when $\sigma_B = \sigma_{CAP-I}$; namely,

$$\rho_{T,B} = 1 - \frac{TE(targ \ et)^2}{2\sigma_B^2} \quad (5)$$

By maximizing the $r(CAP)$ subject to the condition that the variance be identical to the benchmark, and its correlation to the benchmark equal to the target correlation, we find that for mutual fund $I$,

$$a = \frac{\sigma_B^2(1 - \rho_{T,B}^2)}{\sigma_I^2(1 - \rho_{I,B}^2)} = \frac{\sigma_B}{\sigma_I} \sqrt{\frac{(1 - \rho_{T,B}^2)}{(1 - \rho_{I,B}^2)}} \quad (6)$$
\[ b = \rho_{T,B} - a^{*} \frac{\sigma_i}{\sigma_B} \rho_{1,B} \]  \hspace{1cm} (7)

The details of these calculations are provided in Muralidhar (2000). If you substitute for “\(a\)” in equation (7), the allocation to the benchmark is independent of variances and is only a function of the correlation terms. While \(b\) and \((1-a-b)\) may be greater than or less than zero (negative coefficients being equivalent to shorting the futures contract relating to the benchmark and borrowing at the risk-free rate), \(a\) is constrained to being positive as it is not currently possible to short mutual funds. However, with active management strategies, \(a\) can be positive and negative.

This method is preferred to the \(M^2\) as it: (a) expressed risk-adjusted performance in basis points; (b) gives advice on portfolio construction – specifically between the risk-free asset, the benchmark (passive investing) and the active portfolio (active management); and (c) provides rankings that are identical with rankings based on skill for equal time horizons.

Muralidhar (2001) extends this approach to multiple manager portfolio or multiple investment strategies and is briefly summarized here. Define \(K\) as the portfolio of active management strategies, each with weight \(w_i\), and now the objective function is to select optimal weights for each active investment opportunity (or \(a^{*}w_i\)), passive benchmark \((b)\) and risk-free asset \((1-a-b)\), so as to maximize the risk-adjusted return subject to a tracking error target or budget.
Mathematically, this is shown as choosing $w_i$ to

$$
\text{Maximize } r(\text{CAP-K}) = \max \{ a \cdot r(K) + b \cdot r(B) + (1-a-b) \cdot r(F) \} \quad (8)
$$

Subject to

$$
\sigma_{\text{CAP-K}} = \sigma_B \quad (9)
$$

$$
?_{\text{CAP-K}} = ?_{TB} \quad (10)
$$

$$
Sw_i = 1 \quad (11)
$$

The original paper demonstrates that maximizing the $r(\text{CAP})$ as opposed to maximizing the information ratio provides the best outcome as well as outcomes that provide the highest confidence in skill in the strategy. Even where clients are not allowed to “leverage” as Figure 1 demonstrates, some strategies are better delevered to make for optimal portfolio construction. As a result, this technique can be used with any restrictions that a particular client may have.

**A quick numerical simulation**

While the discussion, especially Figure 1, provides adequate reason to disregard the maximization of information ratios as an objective, we demonstrate the same with some numbers. We compare the strategy of maximizing the risk-adjusted return versus the process of either maximizing the information ratio on the active component or the classic two-stage process of differentiating between an active and passive component and then
maximizing information ratios within the active component. We assume that there are three active strategies to be chosen, and their performance relative to the benchmark is provided in Table 1. We also assume a risk-free asset (F) with an annualized return of 5% with no variance. In Table 1, we also provide information on the correlation across investment strategies and also information ratios and risk-adjusted performance (using the single active strategy) assuming a target tracking error of 10%. Notice also that all managers have a tracking error far in excess of the target. Table 1 demonstrates how Manager A, with a negative information ratio has positive risk-adjusted performance (higher than that of Manager B who has a positive information ratio). While not reported in the Table, Manager A has the highest r(CAP) of 35.1%, followed by Manager C with an r(CAP) of 34.1%. This shows that if an investor normalizes for standard deviation or tracking error in the correct manner to determine the highest risk-adjusted return, the information ratio conveys little information about the value or skill of a manager. For simplicity, we do not impose any constraints on the sign or size of any allocation as this would provide a very specific example of a more general result that we are trying to prove.

The more interesting aspect is when there are multiple investment opportunities included in a portfolio. In Table 2, we construct two strategies: (a) maximizing the risk-adjusted performance versus the target tracking error; and (b) maximizing the information of the active component in isolation and then combining with cash and passive benchmarks in
the second stage. The second simulation is conducted in two ways: (i) that we try to maximize the information ratio on the active portion only assuming the tracking error budget of 10% (which is not achievable in this case for the investment options chosen); and (ii) maximize the information ratio on double the budget (20% tracking error) and then decide between this active portfolio and passive management and cash. As Table 2 shows, if we try to maximize the information ratio on the active-only portion, the lowest possible tracking error achievable is 12.7%, which gives a negative information ratio and a poor overall return. Compare this to the situation where the risk-adjusted performance is maximized. This is not a surprise as all the active managers are high tracking error managers. More important, the allocation to Manager C is negative and this was the manager with the highest information ratio. In a perverse way, the high information ratio manager is desired to help improve the portfolio characteristics through a short position.

<INSERT TABLE 2 HERE>

In Table 3, we demonstrate the case where the tracking error on the active-only portion is increased to 20% and the active portfolio allocations are locked in. In this case, the total return is more than that of the maximize risk-adjusted performance simulation, but with higher risk (both absolute and relative). When this active allocation is combined with the risk-free asset and the benchmark, the overall return collapses in order to achieve the tracking error target. In short, maximizing information ratios on active components, either in isolation or a precursor to active-passive allocation decisions for a tracking error budget cannot do as well as strategies that maximize risk-adjusted performance.
Conclusion

Research on risk-adjusted performance measures is providing guidance on optimal portfolio construction for given risk budgets. We have demonstrated that with respect to information ratios on active portfolios, either in choosing a single strategy/manager or in constructing portfolios based on multiple active strategies, bigger is not necessarily better. A naïve approach of maximizing the information ratio across many active alternatives or even maximizing the excess return subject to a tracking error budget suffers from either excluding certain investment alternatives (leveraging or deleveraging using cash and the passive benchmark) or using risk budgets inefficiently so that the chosen alternative may not have the highest return or confidence in skill level. Instead, clients with a target tracking error relative to a benchmark should maximize the risk-adjusted performance of the portfolio through optimal allocations to active, passive and risk-free alternatives while ensuring that the standard deviation of returns is equal to that of the benchmark and the correlation of the returns equal the target correlation. While this in effect leads to the highest possible information ratio on the overall portfolio, it does not require the highest information ratio on the active component as suggested by previous research. This calls for a paradigm shift and a more open approach to optimal portfolio construction.
Figure 1: An evaluation of the M² measure
Table 1: Information on investment strategies

<table>
<thead>
<tr>
<th></th>
<th>Return</th>
<th>Risk</th>
<th>Rf</th>
<th>Bench</th>
<th>Mgr A</th>
<th>Mgr B</th>
<th>Mgr C</th>
<th>Excess Ret</th>
<th>Tracking Err</th>
<th>IR</th>
<th>rRAP</th>
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<tbody>
<tr>
<td>Risk Free</td>
<td>5.00%</td>
<td>0.00%</td>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-18.01%</td>
<td>13.39%</td>
<td>-1.35</td>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
<td>23.01%</td>
<td>13.39%</td>
<td>0</td>
<td>1.00</td>
<td>-0.18</td>
<td>0.36</td>
<td>0.14</td>
<td>0.00%</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mgr A</td>
<td>22.07%</td>
<td>10.88%</td>
<td>0</td>
<td>-0.18</td>
<td>1.00</td>
<td>0.29</td>
<td>0.78</td>
<td>0.94%</td>
<td>18.71%</td>
<td>-0.05</td>
<td>26.0%</td>
</tr>
<tr>
<td>Mgr B</td>
<td>25.02%</td>
<td>15.71%</td>
<td>0</td>
<td>0.36</td>
<td>0.29</td>
<td>1.00</td>
<td>0.82</td>
<td>2.01%</td>
<td>16.58%</td>
<td>0.12</td>
<td>22.1%</td>
</tr>
<tr>
<td>Mgr C</td>
<td>34.44%</td>
<td>15.58%</td>
<td>0</td>
<td>0.14</td>
<td>0.78</td>
<td>0.82</td>
<td>1.00</td>
<td>11.43%</td>
<td>19.02%</td>
<td>0.60</td>
<td>30.3%</td>
</tr>
</tbody>
</table>

Table: Correlation
<table>
<thead>
<tr>
<th></th>
<th>Max r(CAP)</th>
<th>Max IR (Active only)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk free</td>
<td>-73.2%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Benchmark</td>
<td>80.1%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Active (Of which)</td>
<td>93.1%</td>
<td>100.0%</td>
</tr>
<tr>
<td>Mgr A</td>
<td>92%</td>
<td>120%</td>
</tr>
<tr>
<td>Mgr B</td>
<td>14%</td>
<td>156%</td>
</tr>
<tr>
<td>Mgr C</td>
<td>-6%</td>
<td>-175%</td>
</tr>
<tr>
<td>Total</td>
<td>100.00%</td>
<td>100.00%</td>
</tr>
<tr>
<td>Return</td>
<td>36.48%</td>
<td>4.99%</td>
</tr>
<tr>
<td>Risk</td>
<td>13.39%</td>
<td>6.97%</td>
</tr>
<tr>
<td>Tracking Error</td>
<td>10.00%</td>
<td>12.70%</td>
</tr>
<tr>
<td>IR</td>
<td>1.35</td>
<td>-1.42</td>
</tr>
<tr>
<td>Correlation with BM</td>
<td>0.72</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Table 2: Allocations for maximizing risk-adjusted performance versus maximizing information ratios
Table 3: Allocations for maximizing risk-adjusted performance versus maximizing information ratios through a two-stage process (active-only for a 20% budget and then active-versus passive)

<table>
<thead>
<tr>
<th></th>
<th>Max r(CAP)</th>
<th>Max IR (Active only)</th>
<th>Max Portfolio Return (based on active portfolio)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk free</td>
<td>-73.2%</td>
<td>0.0%</td>
<td>-8.8%</td>
</tr>
<tr>
<td>Benchmark</td>
<td>80.1%</td>
<td>0.0%</td>
<td>117.1%</td>
</tr>
<tr>
<td>Active (Of which)</td>
<td>93.1%</td>
<td>100.0%</td>
<td>-8.3%</td>
</tr>
<tr>
<td>Mgr A</td>
<td>92%</td>
<td>-33%</td>
<td>3%</td>
</tr>
<tr>
<td>Mgr B</td>
<td>14%</td>
<td>6%</td>
<td>0%</td>
</tr>
<tr>
<td>Mgr C</td>
<td>-6%</td>
<td>127%</td>
<td>-11%</td>
</tr>
<tr>
<td>Total</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
</tr>
<tr>
<td>Return</td>
<td>36.48%</td>
<td>37.95%</td>
<td>23.36%</td>
</tr>
<tr>
<td>Risk</td>
<td>13.39%</td>
<td>17.98%</td>
<td>14.66%</td>
</tr>
<tr>
<td>Tracking Error</td>
<td>10.00%</td>
<td><strong>20.00%</strong></td>
<td>10.00%</td>
</tr>
<tr>
<td>IR</td>
<td>1.35</td>
<td>0.75</td>
<td>0.04</td>
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<tr>
<td>Correlation with BM</td>
<td>0.72</td>
<td>0.21</td>
<td>0.75</td>
</tr>
</tbody>
</table>
Appendix 1 – Defining Tracking Error and Information Ratio

Define $I$ as the active manager or asset allocation strategy, $B$ as the benchmark, $r$ denotes the return, $\sigma$ denotes the standard deviation, and $\rho$ is the correlation of returns between the manager and the benchmark.

If the tracking error of portfolio 1 versus the benchmark is defined as the standard deviation of excess returns, it is trivial to define $TE(1)$ as follows:

$$TE(1) = \sqrt{\sigma^2_I - 2\rho \sigma_I \sigma_B + \sigma^2_B} \quad (A.1)$$

The annualized information ratio is equal to the annualized excess return divided by the annualized tracking error or, alternatively,

$$IR(1) = \frac{[r(1)-r(B)]}{TE(1)} \quad (A.2)$$
Bibliography


My debt of gratitude to Prof. Franco Modigliani is immeasurable for the hours spent on discussing this topic. My thanks also to Arjan Berkelaar for helpful discussions and finding errors in my work. Thanks also to Sanjay Muralidhar, Ronald van der Wouden, Kenneth Miranda, Barton Waring and Larry Seigel. All errors are mine.

The path breaking work referred to in all the research is Grinold and Kahn (1999). The current papers use this book as the starting point for their discussions. See for example, Blitz and Hottinga (2001). Mashayekhi and Muralidhar (1996) makes this assumption in the context of implementing currency management strategies.

Where the risk-free rate has no zero variance, the denominator is that standard deviation of the portfolio.

Thanks to Ronald van der Wouden for this point.

Many papers adjust for this by allowing for a subjective allocation of confidence in skill (Black and Litterman 1992) or using other measures to convey such confidence (Lam and Lee 2001). Muralidhar (2002) uses an explicit measure of skill to create a time-adjusted “expected” risk-adjusted performance measure.

The author’s experience as a plan sponsor and an asset manager has shown that no guideline ever prevents an active strategy from using cash or the passive benchmark to improve risk-adjusted performance. It is just a bad assumption made by active managers who focused on the wrong objective function; namely, to maximize the information ratio.

In addition, when benchmark returns are negative (e.g., in currency mandates), the $M^2$ measure will incorrectly rank underperforming funds with high volatility are preferred to funds with low volatility. This is a quirk of the method as it is generally assumed that benchmarks have positive returns.

These are heroic assumptions to say the least. Some forecast needs to be made on expected outperformance, variability of performance to achieve this outperformance and correlations between portfolio and benchmark returns. Historical performance is one way of making forecasts, but the M-3 measure is independent of the forecasting technique. In addition, one must believe that markets are inefficient to conduct such analyses.

This measure is independent of the level of tracking error and hence is applicable across all tracking error targets.

This may change with the development of exchange-traded funds (ETFs) on active portfolios. In some cases it may be difficult to short the benchmark as well and then $b$ will need to constrained to being greater than or equal to zero. This would not change the analysis of the measure. In general though, most benchmarks can be shorted either through their futures contract or through a swap.

See also Waring and Castile (1998) for the naïve approach of maximizing multi-manager returns subject to a risk budget.

There are issues about the stability of correlations and standard deviations that have been addressed in previous research and since the goal of this paper is to change the paradigm first, we do not address these issues again in this research.